

Pre-Calculus

Summer Math Pack!

Directions: Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers.

1. For each polynomial function given here, determine:

- i) how many zeros does the function have?
- ii) to the nearest tenth, what is the largest zero?

a) $f(x) = 3x^4 + 4x^3 - x + 0.05$

b) $g(x) = -0.02x^4 + 0.96x^3 + 2.02x^2 - 0.98x - 1$

2. Find an equation of the line passing through the point $A(4, -7)$ and perpendicular to the line through the points $B(-1, -2)$ and $C(5, 8)$.

3. An experiment yields the data shown in the table below:

t (hours)	0	1	2	3	4	5
p (pressure)	6	6.37	6.75	7.16	7.58	8.02

- a) Explain how you know that this data is not linear.
- b) Write the equation of the linear regression function that might be used to model data.
- c) Using your linear model function, estimate the pressure when $t = 8.5$ hours.

4. a) If $f(x) = \sqrt{x^2 + 8x}$, find the domain of function f . Then write a formula for the function whose graph is obtained by:
- b) reflecting the graph of f across the y -axis;
- c) shifting the original graph of f to the left 3 units, flipping the graph over the x -axis, and then shifting it up 4 units.

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5. A function f is given by the formula $f(x) = \frac{R}{x + K}$, where R and K are constants. If the points $A(3, 2)$ and $B(5, 1)$ both lie on the graph of f , determine the numbers R and K **exactly**.

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6. If f is a polynomial function of degree 3 with the numbers 4, 5, and -6 as zeros, and if $f(1) = -210$, determine the formula for $f(x)$.

7. A rational function g has the lines $x = 2$ and $x = -2$ as vertical asymptotes, the line $y = 4$ as a horizontal asymptote, and the numbers 3 and 1 as zeros. Find a formula for $g(x)$.
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8. Given the function $h(x) = \frac{3x}{x+5}$:
- Explain how you know that h has an inverse function.
 - Find a formula for $h^{-1}(x)$.
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9. The half-life of a toxic substance is 11,250 years.
- If 153 gm of the substance is present now, write a formula for the function that gives the amount $A(t)$ that will be present t years from now.
 - When will only 0.1 gm of this substance remain?

10. a) Solve for x : $\log_2(3x - 1) \leq 4$
b) If $f(x) = \ln(x^2 + 4x)$, write a formula for the function g whose graph is obtained by reflecting the graph of f across the y -axis and then shifting it up 3 units and to the left 7 units.

11. Label each statement **TRUE** or **FALSE**.

- a) $10^{\log(5+x^2)} = 5 + x^2$
b) $\ln(A + B) = (\ln A)(\ln B)$
c) $\ln\left(\frac{A}{B}\right) = \ln(A) + \ln\left(\frac{1}{B}\right)$
d) $e^{-\ln(C)} = -C$
e) $(\log_b(x))^y = y \log_b(x)$

12. Label each statement **TRUE** or **FALSE**.

- a) $\sin(x^2) + \cos(x^2) = 1$
b) $\sin^2(x) + \cos^2(x) = 2$
c) $\cos(x + y) = \cos x + \cos y$
d) $\tan(2x) = 2 \tan x$
e) $\sec x = \frac{1}{\sin x}$

13. Solve for x : $5 + \ln x = \frac{14}{\ln x}$

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14. Dave invests \$100 at 8% interest per year. How much does Dave have after 6 years if the interest is
- a) compounded annually
 - b) b) compounded quarterly
 - c) c) compounded continuously.

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15. Describe in words the graph of $y = 2^x$. (Hint: Talk about intercepts, asymptotes, increasing/decreasing, etc.)

16. For the following functions, write down an inverse function and verify that your answer is correct.

a) $f(x) = \frac{x+3}{3}$

b) $f(x) = \sqrt[3]{x-1}$

17. Let $f(x) = \frac{ax+b}{cx-a}$, where a , b , and c are positive constants. Show that f is one-to-one and f is its own inverse function.

18. Solve **exactly** for x : $\log_2(6-x) + \log_2(2-x) = 5$

19. Solve **exactly** for x : $x^{\ln x} = e^{100}$

20. Let $f(x) = \frac{x}{2x-3}$ and $h(x) = \frac{2}{x-1}$.

- Determine a formula for the inverse function, $f^{-1}(x)$.
- Write a simplified formula for the composite function $[h \circ f](x)$.
- Determine the domain of $h \circ f$.

21. Suppose f and g are inverses of each other. What is true about their composition(s)?

22. Suppose $f(x) = x^2 - 1$ and $g(x) = \sin x$. Find

$$(f \circ g)(x), (g \circ f)(x), h(x) = [(f \circ g) \circ (g \circ f)](x).$$

In problems 23-25, solve the inequality.

23) $4x < 2x + 1 < 3x + 2$

24) $2x^2 + x \leq 1$

25) $\frac{2x+1}{x-5} \leq 1$